

**BUNKER HILL
COMMUNITY
COLLEGE**

Division of Liberal Arts and Sciences

CALCULUS I

MAT 281 - ES

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LEARNING PROCEDURE

You are participating in an independent learning course. Your primary goal is to learn Calculus. You will use the resources available to you to do a series of home self-tests that you can mail back to Bunker Hill Community College (B.H.C.C. External Studies Office, 250 New Rutherford Avenue, Boston, Massachusetts 02129-2991) OR hand delivered to the External Studies desk in the Center for Self-Directed Learning (E235). The corrected and graded Home Tests will be returned to you so that you may use them to prepare for the exams. Your presence at BHCC will be required for the taking of EXAMS only. The EXAMS will be taken at the CSDL and can be obtained at the External Studies desk. The exams are '**closed book**' and ALL scrap work must be attached to the answer sheet! Attach solutions to home self-tests as well!

WARNING! Graphing calculators used during an EXAM will have the memory erased before starting and upon completion of the EXAM. Be sure to make a back-up of material you wish to keep in memory at home.

I will guide your progress by providing a detailed syllabus, to include text sections and suggested exercises to study. You will decide how many of the suggested exercises you need to do in order to understand the concept. Make sure you try some problems from each new set of directions found in the list of suggested exercises. The answers to the odd questions are in the back of your text. The solutions to ALL odd exercises are available in the optional Solutions Guide. I will attempt to put notes on your Home Tests regarding mistakes before returning them to you. The suggested exercises from the Review sections of the book will provide excellent preparation for the EXAMS.

DO NOT send your work on the suggested exercises for me to correct.

Exams will **NOT** be given until all of the appropriate Take-Home quizzes have been corrected. This policy is intended to ensure that any mistakes made on the Take-Home quizzes will not be made again on the Exams. **You will need to produce a picture ID in order to take the Exams.**

You may contact me directly by telephone, (617-228-2268), and use my voice mail. I will return calls as soon as possible. E-mail may be more convenient and faster. rtcarlson@bhcc.mass.edu

Independent learning does NOT mean open-ended learning. You need to establish a reasonable schedule taking into account delays in the mailing process and allowing me at least a week to correct your Home Tests. In order to speed up this process, I suggest you let me know when you deliver each Home Test and/or take an EXAM. This will inform me that paperwork is to be expected for evaluation. **PLEASE**, do not save up multiple Home Tests for evaluation. The understanding developed for each assignment supports the next one.

Procrastination creates the biggest obstacle to your success in this course. Get started NOW! I look forward to assisting you with your learning of CALCULUS.

H.

COURSE OUTLINE

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Lesson	Home Test	Chapter	Text Sections
1.	Take-Home EXAM	Prerequisites	selections from ALL
2.	A.	1	1.1; 1.2; 1.3
	B.	1	1.4; 1.5
3.	C.	2.	2.1; 2.2; 2.3
	D.	2.	2.4; 2.5; 2.6
4.	MID EXAM	1 and 2	selections from review exercises
5.	E.	3.	3.1; 3.3; 3.4
	F.	3.	3.5; 3.6
	G.	3.	3.7; 3.9; Appendix G
6.	H.	5.	5.1; 5.3; 5.4(derivatives only) 5.5; 5.8
7.	FINAL EXAM	3 and 5 only	selections from review exercises

I.

GRADING POLICY

MAT281

EXAMS will be graded on a 0 - 100 scale.

Home Tests will be marked on a 0 - 100 scale also. However, each Home Test will be assigned a number from 0 to 3 according to the following;

3 points earned for [83 - 100],

2 points earned for [70 - 83),

1 point earned for [50 - 70), and

0 points earned for [0 - 50).

The total points earned for the 8 Home Tests will represent the % assigned to the home portion of the final grade.

PRE-EXAM	=	10%
MID-EXAM	=	31%
FINAL EXAM	=	35%
HOME TESTS	=	24%
		<hr/>
		100%

Final grades are calculated as per the college catalogue.

[0 - 60); F
[60 - 70); D
[70 - 77); C
[77 - 80); C+
[80 - 83); B-
[83 - 87); B
[87 - 90); B+
[90 - 94); A-
[94 - 100]; A

CHAPTER 2

A. RATIONALE

Calculus represents the culmination of years of basic mathematical preparation. Calculus puts to use your knowledge of arithmetic, algebra and trigonometry. Calculus allows you to explore problems that could not be solved by conventional means. Calculus also is a beginning. The applications of Calculus to business, manufacturing, computers, psychology and the sciences are endless. Most importantly however, you are learning Calculus in order to become a problem solver. The techniques developed through Calculus regarding accepting a problem, analyzing it, devising a method of solution and finally solving the problem become a solid foundation that can be applied to almost any level of employment.

You will need to apply your previous knowledge to new situations and apply new knowledge to old situations. You are becoming a more sophisticated user of mathematics. Eventually you will apply your understanding of Calculus to the real life surrounding you.

RATIONALE:

The ability to use Calculus efficiently depends heavily on your knowledge of algebra and trigonometry. Calculus is a tool that expands your ability to analyze mathematical problems. You will be using visual and graphical references throughout the course. That is why a strong review of functions and graphing concepts is necessary. Techniques applied to simplifying and solving algebraic and trigonometric functions are **assumed** to be a **strong** part of your prior experience. Please make sure that is the case with you.

GOALS: By studying this lesson you will ...

- ♣ recall properties of Real numbers.
- ♣ explore methods of graphing equations.
- ♣ recall specific properties of lines in the plane.
- ♣ absorb the concept of functions.
- ♣ reestablish basic understanding of trigonometry.(see appendix D)
- ♣ fit models to data using a graphing calculator.

OBJECTIVES: You will ...

- ♠ plot data on a cartesian graph.
- ♠ match equations to graphs.
- ♠ find intercepts.
- ♠ check points that may lie on the graph of an equation.
- ♠ sketch graphs of equations.
- ♠ find points of intersection.
- ♠ find slope and y-intercept.
- ♠ write the equation of a line using ...
 - a) two points
 - b) one point and the slope.
- ♠ write equations of lines parallel to a given line.
- ♠ write equations of lines perpendicular to a given line.
- ♠ evaluate a function at a given value.
- ♠ identify y as a function of x .
- ♠ find composite functions.
- ♠ find Real zeros of a function.
- ♠ write functions from word problems.
- ♠ solve those word problems.
- ♠ fit a linear model to data.
- ♠ fit a quadratic model to data.
- ♠ fit a trigonometric model to data.
- ♠ review Real numbers and the Real number line. (see Appendix D)
- ♠ review Trigonometric Functions. (see Appendix D)
- ♠ review the Cartesian Plane. (see Appendix D)

Note: Appendix D is found at www.college.hmco.com

LEARNING ACTIVITIES: Study Prerequisite Chapter.

section	page	suggested exercises
P.1	8	5-12;17-24;37-54;63-72;73-75;77-78
P.2	16	9-14;21,23-42; 49-56; 59-70
P.3	27	1-16; 33-36; 52; 71,72
P.4	33	1-10
Review Chapter P	36	1-4;7-14;17,18,21-32,33-36;39,40,48

Try to complete the Take-Home PRE-EXAM within three weeks of starting this course.

ASSESSMENT/RESOURCES: See Chapter 1.

LESSON 2

LIMITS and THEIR PROPERTIES

RATIONALE:

Dilemmas surrounding ancient problems, where algebra failed to satisfactorily explain solutions, were very frustrating to past mathematicians. The birth of calculus required a certain *leap of faith*; an acceptance of a definition for *closeness*. This idea was called Limit. The concept of Limit had to be understood and organized in order for the rest of calculus to work. You must also begin by developing your understanding of the concept of Limit so you can connect calculus to your previous mathematical experiences. Thus you will open up the power of calculus as a tool for problem solving.

GOALS: By studying this lesson you will ...

- ♣ embrace the concept of Limit.
- ♣ apply the properties of Limits to the evaluation of Limits.
- ♣ understand continuity.
- ♣ recognize infinite Limits.

OBJECTIVES:

For Assignment A, you will ...

- ♠ approximate length and area by repeated sums.
- ♠ find the limit of an algebraic function.
- ♠ find the limit of a trigonometric function.
- ♠ find the limit of functions that begin as indeterminate.
- ♠ find the limit of special trigonometric functions.
- ♠ use the limit definition of velocity to solve falling object problems.

For Assignment B, you will ...

- ♠ find one-sided limits of functions.
- ♠ determine points of discontinuity.
- ♠ identify discontinuities as *removable* or *non-removable*.
- ♠ use interval notation to identify the domain of a function.
- ♠ apply the Intermediate Value Theorem to the verification of the guaranteed value.
- ♠ find vertical asymptotes of functions.
- ♠ find one-sided infinite limits of functions.
- ♠ apply limits to the solving of word problems.

LEARNING ACTIVITIES:

For Assignment A: section	Study sections page	1.1, 1.2 and 1.3 . suggested exercises
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1.1	47	10-11
1.2	54	9-20; 23-38, (find L only)
1.3	65	5-22; 27-36; 49-62; 67-78; 83-86; 101-104

Do HOME TEST A. Deliver answers and work to BHCC.

For Assignment B:		Study sections page	1.4 and 1.5 . suggested exercises
1.4	76		7-20; 33-52; 69-72; 75-78; 79-82; 83-86
1.5	85		1-4; 5-8; 9-28; 33-48; 49-52

Do HOME TEST B. Deliver answers and work to BHCC.

ASSESSMENT/RESOURCES: See Chapter 1.

RATIONALE:

Derivatives form the basis of the rest of Calculus. Formulas need to be developed in order to make solving real problems easier. The idea that a derivative must be related to an original function should be clear from the word itself. The relationship can be interpreted in different ways. Slopes of curves and rates of change are the key interpretations.

GOALS: By studying this lesson you will ...

- ♣ understand the connection between slope and the derivative.
- ♣ recognize rules for differentiation.
- ♣ recognize implicit functions.
- ♣ establish the derivative as a representation of a rate of change.

OBJECTIVES:

For Assignment C, you will ...

- ♠ use the definition of the derivative to find $f'(x)$.
- ♠ find the equation of a tangent line to a graph of f at a point.
- ♠ find the equation of a line tangent to f and parallel to a given line.
- ♠ use the Alternative form of the derivative to find $f'(c)$.
- ♠ find values of x where $f(x)$ is differentiable.
- ♠ find the derivative of a given function.
- ♠ evaluate the derivative of a function.
- ♠ find the equation of a tangent line.
- ♠ find the average rate of change.
- ♠ solve falling body problems.
- ♠ solve rate of change word problems.
- ♠ differentiate algebraic functions.
- ♠ differentiate trigonometric functions.
- ♠ use power rule, product rule and quotient rule as needed.
- ♠ find the second derivative.
- ♠ find higher order derivatives.
- ♠ solve velocity or acceleration problems.

For Assignment D, you will apply the chain rule to ...

- ♠ find the derivative of composite algebraic functions.
- ♠ find the derivative of composite trigonometric functions.
- ♠ evaluate the derivative of composite functions.
- ♠ solve word problems.

you will ...

- ♠ find dy/dx by implicit differentiation.
- ♠ evaluate dy/dx at a given point.
- ♠ find the second derivative in terms of x and y .
- ♠ find the equations of tangent and normal lines to circles.
- ♠ find the values of dx/dt or dy/dt .
- ♠ find related rate word problems for:
 - 1) conic section forms,
 - 2) right triangle forms,
 - 3) angles of elevation,
 - 4) linear vs angular speed.

LEARNING ACTIVITIES: Lesson 3

section	page	suggested exercises

For Assignment C:	Study sections	2.1; 2.2 and 2.3.
2.1	101	5-36; 39-42; 61-64;71,72
2.2	113	3-62;75,76, 87-94;99,103
2.3	124	1-36; 39-54; 59-70; 75-80; 83-92; 101,103
Do HOME TEST C.		Deliver answers and work to BHCC.
For Assignment D:	Study sections	2.4; 2.5; and 2.6 .
2.4	133	7-34; 35-44(by hand) 47-78; 79-82 91-94; 105-108
2.5	142	1-16;21-28; 35-40;43,44,47,48
2.6	149	1-4;13-27; 30-32;35,36, 43-47
Do HOME TEST D.		Deliver answers and work to BHCC.

ASSESSMENT/RESOURCES: See Chapter 1.

LEARNING ACTIVITIES:

Study chapter 1 and 2 reviews.

section	page	suggested exercises
ch. 1 review {7-10}	88	11-22; 29-30; 31-36; 38-44; 53-68 find a delta using limit definition for extra credit
ch. 2 review	153	1-4; 9-12, 17-35; 41-62; 65-80; 81-88, (by hand); 89-92; 97,98 99-106; 107-110

Schedule at least 2½ hours for the MID-EXAM at CSDL.**ASSESSMENT/RESOURCES:**

See Chapter 1.

RATIONALE:

As the title suggests, applications of the derivative will establish some common uses. Curve analysis and optimization problems are popular uses of the derivative. Graphing calculators or software will produce sketches of graphs for you, but understanding the relationship of the components that produce the graphs will be useful in analysis.

GOALS: By studying this lesson you will ...

- ♣ use the menus of your graphing calculator to find intercepts, extrema, and points of inflection.
- ♣ compare the extrema of a function.
- ♣ recognize critical numbers.
- ♣ search for characteristics of parts of graphs.
- ♣ search for asymptotes.
- ♣ analyze graphs of functions.
- ♣ understand applications of derivatives to minimum/maximum problems.
- ♣ realize that derivatives can be used to approximate.

OBJECTIVES:

For Assignment E, you will ...

- ♣ find critical numbers.
- ♣ locate absolute extrema.
- ♣ solve word problems related to extrema.
- ♣ locate intervals where a function is increasing or decreasing.
- ♣ determine the effect the relationship between f and g has on f' and g' .
- ♣ find relative extrema.
- ♣ find points of inflection.
- ♣ sketch graphs, identifying intercepts.
- ♣ solve word problems using relative extrema.

For Assignment F, you will ...

- ♣ find limits as x goes to ∞ .
- ♣ sketch graphs that may have asymptotes.
- ♣ sketch graphs of algebraic functions.
- ♣ sketch graphs of trigonometric functions.
- ♣ identify intercepts, extrema and points of inflection
- ♣ find any vertical, horizontal and/or slant asymptotes.

For Assignment G, you will ...

- ♣ solve minimum/ maximum word problems of type:
 - 1) sum or product of numbers,
 - 2) area or volume.
- ♣ solve *Business applications* (See Appendix G)
- ♣ find the differential dy .
- ♣ use differentials to approximate error.
- ♣ use differentials to approximate roots or powers.

LEARNING ACTIVITIES:

Lesson 5

section	page	suggested exercises
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NOTE: Exercises that say "use a computer algebra system" should be done using your graphing calculator and/or by hand.

For Assignment E: Study sections 3.1; 3.3 and 3.4 .

3.1	165	11-32; 37-40; 55,58
3.3	181	1-10;11-36; 49-54; 62,63
3.4	189	1-40; 53-56; 67-69

Do HOME TEST E. Deliver answers and work to CSDL.

For assignment F: Study sections 3.5 and 3.6 .

3.5	199	15-34; 49-78;82-84
3.6	208	7-50; 63,64

Do HOME TEST F. Deliver answers and work to CSDL.

For Assignment G: Study sections 3.7 and 3.9 and appendix G.

3.7	216	2-14,17,18,20,23,29,30,33,34,54,58,59
3.9	233	7-20; 29-35; 45-48;
Appendix G	www.college.hmco.com	look especially at ex. 7 on p.232 3-16; 18,21,23,24,25

Do HOME TEST G. Deliver answers and work to CSDL.

ASSESSMENT/RESOURCES:

See Chapter 1.

LESSON 6

Transcendental Functions

RATIONALE:

There is another class of functions beyond algebraic and trigonometric. So far all of your variables have existed as bases. You need to explore what happens when the variable becomes an exponent. This occurrence leads to the need for logarithms to help with the solution of problems. Additionally, since much of our mathematical system must work in forward or reverse, you need to deal with inverse functions and their applications as well. These functions can be differentiated. Therefore, these functions will help you complete your study of derivatives and make the solution of special problems possible.

GOALS: By studying this lesson you will ...

- ♣ understand the relationship of e to logarithms.
- ♣ recognize when to use logarithmic differentiation.
- ♣ understand the concept of a monotonic function.
- ♣ recognize inverse functions.
- ♣ understand the uniqueness of the natural exponential function.
- ♣ recognize inverse trigonometric functions.

OBJECTIVES:

For Assignment H, you will ...

- ♠ use properties of logarithms to rewrite natural log expressions.
- ♠ find the derivatives of \ln functions.
- ♠ use logarithmic differentiation to find dy/dx .
- ♠ find the inverse of a given function.
- ♠ show that a function may be monotonic.
- ♠ write an exponential equation as a logarithmic equation or vice-versa.
- ♠ find the derivative of a function containing e^u .
- ♠ find the derivative of a function with x as the exponent.
- ♠ use logarithmic differentiation to find dy/dx .
- ♠ express inverse trigonometric expressions in algebraic form.
- ♠ find derivatives of inverse trigonometric functions.

LEARNING ACTIVITIES: Lesson 6

section	page	suggested exercises

For Assignment H:	Study sections 5.1, 5.3, 5.4, 5.5, 5.8.	
5.1(from Theorem 5.1)	321	11-16;19-34;45-60;87-92
5.3	338	23-30; 47-52
5.4(p.341-344)	347	39-60
5.5(skip ex.4,p.354)	357	41-60; 79-92
5.8	386	5-28; 41-43;45-49;ex. 7 p384

Do HOME TEST H.

Deliver answers and work to BHCC.

ASSESSMENT/RESOURCES:

See Chapter 1.

LESSON 7

FINAL EXAM

LEARNING ACTIVITIES:

Study chapter 3 and 5 reviews.

section	page	suggested exercises
ch. 3 review	235	15-20; 23-26; 33-44; 45-62
Use your graphing calculator!!		65,68,75,76; 81-84
ch. 5 review	405	3-6; 9-12; 25,26; 39-48;
		65-68; 75-76; 91-97 ex. 7 p384

Schedule at least 2½ hours at CSDL to take the FINAL EXAM.

ASSESSMENT/RESOURCES:

See Chapter 1.

CHAPTER 3

BIBLIOGRAPHY and REFERENCES

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Calculus 7/th. Boston, Massachusetts:
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Texas Instruments, TI-8* Graphics Calculator, Website: www.TI.com

CHAPTER 4

HOME TESTS

*Follow the directions found in the
LEARNING ACTIVITIES **before** doing any
of these self-tests.*